

# Optimal multi-floor plant layout based on the mathematical programming and particle swarm optimization

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**Abstract:** In the fields of researches associated with plant layout optimization, the main goal is to minimize the costs of pipelines and pumping between connecting equipment under various constraints. However, what is the lacking of considerations in previous researches is to transform various heuristics or safety regulations into mathematical equations. For example, proper safety distances between equipments have to be complied for preventing dangerous accidents on a complex plant. Moreover, most researches have handled single-floor plant. However, many multi-floor plants have been constructed for the last decade. Therefore, the proper algorithm handling various regulations and multi-floor plant should be developed. In this study, the Mixed Integer Non-Linear Programming (MINLP) problem including safety distances, maintenance spaces, etc. is suggested based on mathematical equations. The objective function is a summation of pipeline and pumping costs. Also, various safety and maintenance issues are transformed into inequality or equality constraints. However, it is really hard to solve this problem due to complex nonlinear constraints. Thus, it is impossible to use conventional MINLP solvers using derivatives of equations. In this study, the Particle Swarm Optimization (PSO) technique is employed. The ethylene oxide plant is illustrated to verify the efficacy of this study.

**Key words:** Plant layout optimization, MINLP, Particle swarm optimization, Multi-floor process

## Introduction

After selecting the type of a process and determining the specifications of all equipment, the next step is to design a plant layout how to determine the location of each process units in an area with significant engineering creativities, heuristics, prior knowledge, and so on<sup>1-4</sup>). Thus, the total construction cost is the combination of the purchasing equipment cost, the piping cost and the site cost. Moreover, a plant layout should secure enough maintenance

and safety spaces for efficient accessibilities and safety requirements to repair process units and prevent domino impacts. In addition, in case of off-shore plants, multi-floor processes have to be installed in the limited site. These issues make the plant layout problems very difficult and complex.

To solve this problem, various methods have been recently developed for the last two decades. Suzuki *et al.*<sup>1</sup>) developed a heuristic rules for the two-dimensional layout problems. Mixed Integer Linear Programming (MILP) models have been employed as considering various sizes and geometries of equipment based on the assumption that all of equipments and connections are rectangular shapes and rectilinear<sup>2, 3</sup>).

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Some researchers have transformed this problem into MILP or Mixed Integer Non-Linear Programming (MINLP) models to find the optimal plant layout via conventional optimization solvers such as General Algebraic Modeling System (GAMS) with the consideration of safety issues<sup>7)</sup>. Castell *et al.*<sup>5)</sup> proposed the genetic algorithm-based method with the Mond Index. Prugh<sup>6)</sup> proposed an MILP model considering Dow's fire and explosion index<sup>8)</sup>. However, since the most previous researches have been focused on only single-floor problems, it is urgently needed to develop an efficient method handling multi-floor plant layouts such as Floating Production, Storage and Offloading (LNG-FPSO). Han *et al.*<sup>9)</sup> proposed the MINLP model with the safety considerations including the impact of possible accidents and the individual risk factors. But, it is really hard to evaluate the objective individual frequencies of accidents and their consequences.

To tackle these difficulties in this study, the safety distance between equipments is fixed as a constant to maintain the sufficient empty spaces for safety, maintenance and repairs. This factor is transformed into inequality or equality constraints. However, it would be possible that there are other constraints according to the type of process and it is not always possible to represent these with mathematical formula. Thus, it is impossible to make this problem suitable mathematical formula for the conventional MINLP solvers which use the derivatives of equations in case the equations are very complex and severe nonlinear.

In this study, the PSO (Particle Swarm Optimization) technique, which is one of the representative sampling approaches, is employed to solve the multi-floor MINLP model. PSO is a population based algorithm introduced by Kennedy *et al.*<sup>10)</sup> which mimics flocks of bird. Many researchers demonstrated that PSO is more efficient than other heuristic optimization methods and is cheaper to implement<sup>11)</sup>.

This paper is organized as follows. In second section, the basic equations to model the plant layout problem are reviewed. In third and fourth section describe the concept of the proposed objective function and an optimization solver. Fifth section discusses the results of the proposed algorithm for ethylene oxide plant. The final section gives concluding remarks.

### The Problem Description

To build a mathematical equations for finding an optimal multi-floor plant layout, the following information have to be fixed previously:

- A set of  $N$  equipment items and their size
- The number of floors, their sizes and heights
- The cost data of pipe lines and pumps
- Process Flow Diagram
- The fixed minimum safety distance between equipment items

Firstly, it is assumed that the shape of all equipments is rectangular and they are allowed to rotate by 90 degrees.  $a_i$  and  $b_i$  are the length of both sides for an equipment,  $i$ . The equation for equipment orientations is as the following:

$$l_i = a_i O_i + b_i (1 - O_i), \forall i \quad (1)$$

$$d_i = a_i + b_i - l_i, \forall i \quad (2)$$

where  $O_i$  is the binary parameter to determine the rotation for equipment item,  $i$ . If the value of  $O_i$  is 1, it means that there is no rotation. Otherwise, i.e. the value of  $O_i$  is 0,  $i$  should be rotated by 90. Based on this equation, the length and width of each equipment are determined.

Next, two binary parameters, which indicate the locations of equipments at a specified floor, are employed. The binary parameter,  $V_{ik}$ , is assigned as 1 if the equipment item  $i$  is installed on  $k$ -th floor; otherwise  $V_{ik}$  would be 0. Another binary parameter  $F_{ij}$  is used to check the floor locations of equipments. If  $F_{ij}$  is 1, two equipments,  $i$  and  $j$ , are on the same floor, otherwise,  $F_{ij}$  is 0 and it means that two equipments are installed on the different floor. In case  $F_{ij}$  is 1, the overlapping problem between  $i$  and  $j$  has to be considered. Otherwise, there is no overlapping between  $i$  and  $j$ .  $NF$  is the total number of floors.

$$F_{ij} = V_{ik} \times V_{jk}, \forall i \quad (3)$$

$$m - (ES_i + ES_j) = C_{ij} \rightarrow \begin{cases} f & C_{ij} \geq 0, S_{ij} = 1 \\ \text{otherwise,} & S_{ij} = 0 \end{cases} \quad (4)$$

$$|x_i - x_j| \geq \left( \frac{l_i + l_j}{2} + (ES_i + ES_j) \times (1 - S_{ij}) + m \times S_{ij} \right) \times F_{ij} \quad (5)$$

$$|y_i - y_j| \geq \left( \frac{d_i + d_j}{2} + (ES_i + ES_j) \times (1 - S_{ij}) + m \times S_{ij} \right) \times F_{ij} \quad (6)$$

for  $\forall i = 1, \dots, N-1, \forall j = i+1, \dots, N, \forall k = 1, \dots, NF$

Since most equipments of onshore or offshore plants have to be repaired onsite, enough maintenance spaces for all equipments should be required. In this study,  $ES_i$  is defined with the maintenance space for equipment,  $i$ , and it is assumed that the maintenance space for each equipment is

set to 30% of each equipment size. Moreover, in order to prevent the propagation of accidents, the minimum safety distances between equipments should be also defined. In this study, the minimum safety distance,  $m$ , is defined and the distances between equipments have to be larger than  $m$ . Throughout the consideration of maintenance spaces and safety distances, the impact of problems due to maintenances, repairs and accidents can be reduced.

Also, the size of floors has to be considered. According to the property of problems, the equations for considering the size can be changed. If the size is fixed previously, the equipments should be located into the area. Otherwise, the objective function should include a term to minimize the land size. These equations can be derived as the following:

$$x_i + \frac{l_i}{2} + ES_i \leq X_{\max}, \quad x_i - \frac{l_i}{2} - ES_i \geq 0 \quad (7)$$

$$y_i + \frac{d_i}{2} + ES_i \leq Y_{\max}, \quad y_i - \frac{d_i}{2} - ES_i \geq 0 \quad (8)$$

The next equations are to calculate the piping distances between equipments  $i$  and  $j$  as avoiding the overlapping and satisfying the constraints. In this study, it is assumed that all connections between the equipment can be evaluated based on the geometry center of equipment. The rectilinear distance has been introduced to consider more realistic piping conditions. The total rectilinear distance between equipments  $i$  and  $j$  is evaluated based on the center of equipments by the considering relative distances in  $x$ ,  $y$ , and  $z$  coordinates.  $H$  and  $NF$  indicate the height floor and the number of floors. The vertical lengths between equipments should be evaluated as considering the differences of floors and their original heights from a ground.  $X_{ij}$  and  $Y_{ij}$  are the linear distances between equipments,  $i$  and  $j$  in  $x$  and  $y$  axis.  $H_i$  is the height of a floor where  $i$  is installed and  $U_i$  is the total height of  $i$  including the height of floor and the center of  $i$  in  $z$  coordinate.  $U_{ij}$  and  $TD_{ij}$  are a vertical distance and the total rectilinear distance between  $i$  and  $j$ .

$$X_{ij} = |x_i - x_j|, Y_{ij} = |y_i - y_j| \quad (9)$$

$$H_i = H \times \sum_{k=1}^{NF} k \times V_{ik}, \quad U_i = H_i + z_i, \quad U_{ij} = |U_i - U_j| \quad (10)$$

$$TD_{ij} = X_{ij} + Y_{ij} + U_{ij} \\ \text{for } \forall i = 1, \dots, N-1, \forall j = i+1, \dots, N \quad (11)$$

Moreover, there are many constraints associated with the working spaces for the maintenances and the passages

for the operators. It is very hard to generalize the model and represent as the mathematical formula, since these conditions vary according to the types and circumstances of plants. However, these constraints should be included. To construct a general model, a penalty function is employed. If the location of equipments item violates these constraints, a large amount of the penalty is assigned to the total costs. This helps the optimal solution to minimize a penalty and to satisfy all assumed constraints.

## The Objective Function

The objective function is the sum of piping, pumping, land and risk and the optimal solution have to satisfy various constraints. In equation (4),  $CC_{ij}$  is the piping cost per unit distance. Also, the pumping cost should be considered, since the vertical pumping cost is much expensive than the horizontal one.  $CH_{ij}$  and  $CV_{ij}$  are the horizontal and vertical pumping cost per unit distance.

In case the size of land is not fixed, the land cost has to be also included in the objective function.  $A$  is the cost per unit square meter.

$$\text{Min.} \sum_i^N \sum_{j \neq i}^N [CC_{ij} \times TD_{ij} + CV_{ij} \times U_{ij} + CH_{ij} (X_{ij} + Y_{ij}) + A \times X_{\max} \times Y_{\max} + p(x, y)] \quad (12)$$

As explained in a previous section, the penalty factor,  $p(x, y)$ , is inserted to find the optimal solution satisfying the various constraints. If there is the violation of constraints, a large amount of penalty would be assigned. This leads the positions of equipments to satisfy all constraints. Design variables of the objective function are as the following:

- $O_i$ : the rotation binary parameter.
- $V_{ik}$ : the floor binary parameter for evaluating the location of equipment.
- $x_i, y_i$ : the position of equipment according to  $X$  and  $Y$  axis.

If the number of equipments is  $n$ , the total number of designed variables should be  $4 \times n$ , respectively.

## Particle Swarm Optimization (PSO)

The locations of each equipment items should be determined to minimize the total costs. As explained in the previous section, there are many constraints for the working spaces and passages, which have many limitations for the mathematical formulations. In addition, the

type of constrains vary according to the floor and the type of a process. Thus, it is not always possible to use conventional tools such as GAMS for solving this problem, since the derivatives of the constraints are not available. As an alternative, PSO (Particle Swarm Optimization) technique is employed in this study. PSO is a population based sampling optimization technique motivated by the social behaviour of collection of animals<sup>10</sup>. It starts with randomly generated swarms, called particles, remember the best solution found. The particles move around the solution space with adjusted velocities and have a tendency to fly towards the global optimal solution over the optimal procedure. The attractive features of PSO are that it does not need to evaluate derivatives of objective function and constraints. Moreover, there are relatively a small number of parameters to adjust<sup>12</sup>. Many researchers have modified PSO to solve the MINLP problem, since the original PSO cannot handle the integer variable.

In this study, decision variable,  $O_i$  and  $V_{ik}$  are integer and the value of these is rounded to the nearest integer based on the original PSO. The detailed steps and advantages of PSO are provided by Schwaab *et al*<sup>12</sup>.

### Case Studies—The First Problem

The proposed algorithm is tested with ethylene oxide (EO) plant in Fig. 1. The EO plant is well-known due to its recent accident histories. This plant consists of EO reactor, EO absorber, the CO<sub>2</sub> absorber, and so on.

It is assumed that two potential floors are available and the floor heights are two types (5 and 7 m). The floor size is fixed as 3,600 m<sup>2</sup> (60 m × 60 m). The minimum safety distance,  $m$ , is fixed as 4 m. Table 1 shows basic data including the all of equipments' connection, pumping costs and each equipment's sizes. This basic data is taken from Han *et al*<sup>9</sup>.

Tables 2 and 3 show the summarizations of the results calculated by PSO. In case the floor height are 5 m and 7 m, the best costs of objective function are  $1.42208 \times 10^5$ , and  $1.80894 \times 10^5$ . Figures 2 and 3 show the best plant layout results satisfying all assumed constraints. According to various tests with many conditions, it is concluded that the vertical pumping cost is the most important factor to determine the plant layout. Figure 4 shows the portion of cost according to piping cost, horizontal and vertical pumping costs.

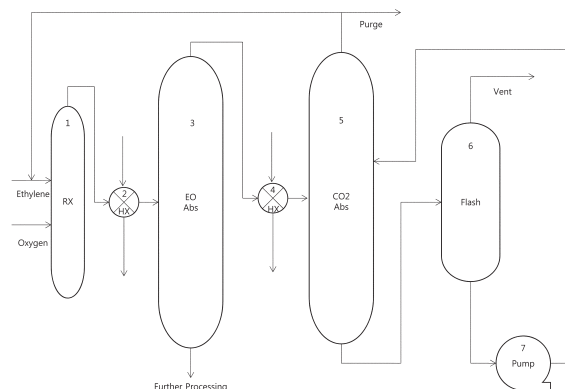


Fig. 1. The PFD of Ethylene Oxide plant.

Table 1. Basic information of the EO plant taken from Han *et al.*<sup>9</sup>

Eq. No.	Dim. a	Dim. b	Cost	Connection	CC <sub>ij</sub> (m)	CH <sub>ij</sub> (m)	CV <sub>ij</sub> (m)
1	5.22	5.22	335,000	2	200	400	4,000
2	11.42	11.42	11,000	3	200	400	4,000
3	7.68	7.68	107,000	4	200	300	3,000
4	8.48	8.48	4,000	5	200	300	3,000
5	7.68	7.68	81,300	1	200	100	1,000
6	2.60	2.60	5,000	6	200	200	2,000
7	2.40	2.40	15,000	5	200	150	1,500

Table 2. The optimal solution in case the height of floor is 5 m

Number	Equipment	floor	x (m)	y (m)
1	Reactor	2nd	24.40	29.70
2	Heat exchanger	2nd	11.09	43.02
3	EO absorber	1st	11.10	42.99
4	Heat exchanger	1st	24.91	29.74
5	CO <sub>2</sub> absorber	2nd	34.85	19.25
6	Flash drum	1st	41.28	19.91
7	Pump	1st	34.77	13.41

Table 3. The optimal solution in case the height of floor is 7 m

Number	Equipment	floor	x (m)	y (m)
1	Reactor	2nd	38.32	14.70
2	Heat exchanger	1st	29.90	14.68
3	EO absorber	1st	45.18	29.96
4	Heat exchanger	2nd	27.35	29.96
5	CO <sub>2</sub> absorber	2nd	14.42	42.91
6	Flash drum	1st	9.083	42.92
7	Pump	1st	15.59	49.43

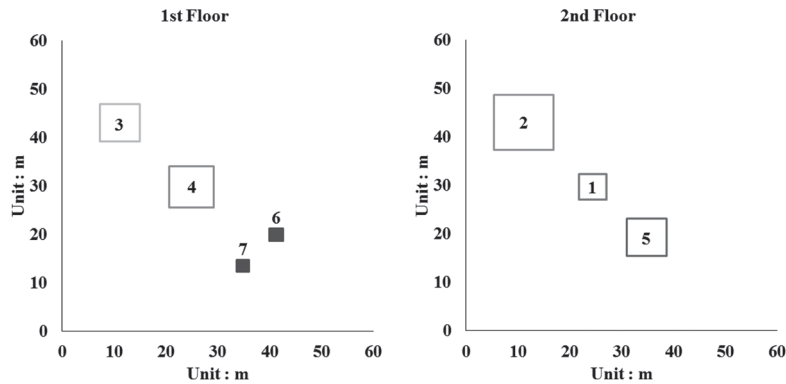


Fig. 2. The results of optimal solution calculated by PSO in case the height of floor is 5 m.

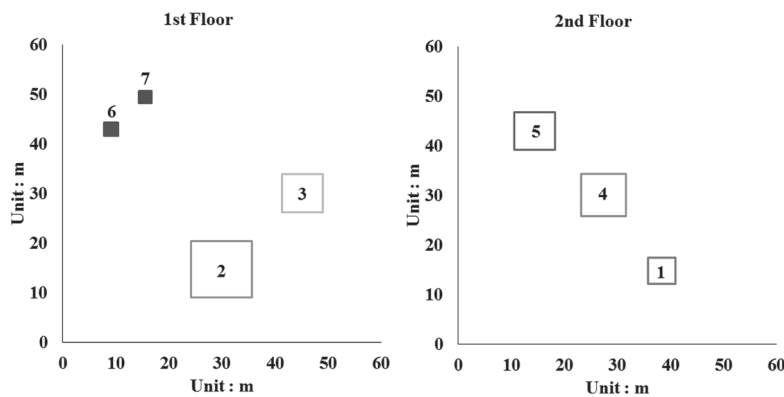


Fig. 3. The results of optimal solution calculated by PSO in case the height of floor is 7 m.

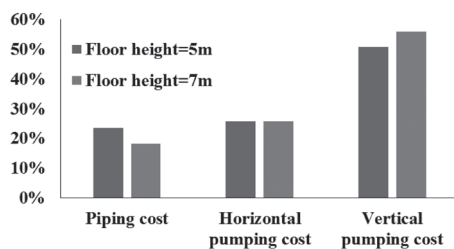


Fig. 4. The cost ratios according to piping cost, horizontal and vertical pumping costs.

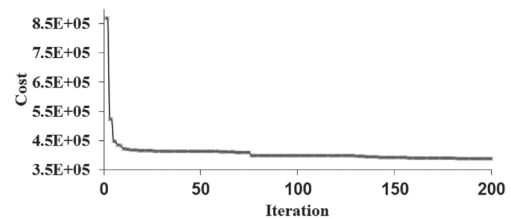


Fig. 5. Minimum cost during PSO iterations.

### Case Studies—The Second Problem

In the next case, only one floor is available and the floor size is not fixed previously. Therefore, the goal of this case is to find the best plant layout with the smallest area, since, the cost of land accounts for the largest portion for building a plant in general. In this study, it is assumed that the land cost per unit square is 26.6<sup>9)</sup>. Figure 5 shows the best results of objective function during PSO iterations. Figure 6 and Table 4 show the best results satisfying all

assumed constraints. In this case, it is verified that the best cost of objective function is  $3.8835 \times 10^5$  and the land cost accounts for the largest portion in Fig. 7. The horizontal and vertical length of a land is 90 m and 94 m.

### Concluding Remarks

To handle a multi-floor optimal layout problem, a MINLP model with the consideration of safety distances, maintenance spaces, multi-floor conditions, etc. is proposed. To make a model, all constraints are transformed into mathematical equations and the objective function is

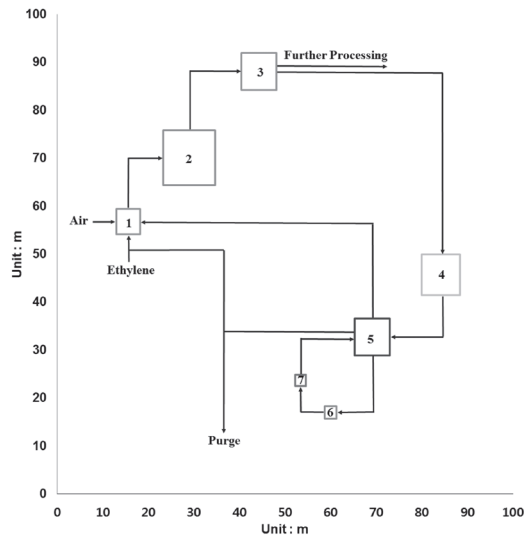


Fig. 6. The optimal solution in case the only one floor is available and the land size is not fixed.

Table 4. The optimal solution in case the only one floor is available and the land size is not fixed

Number	Equipment	$x$ (m)	$y$ (m)
1	Reactor	15.56	56.81
2	Heat exchanger	28.93	70.16
3	EO absorber	44.28	88.09
4	Heat exchanger	84.13	45.70
5	CO <sub>2</sub> absorber	69.11	32.77
6	Flash drum	59.95	17.03
7	Pump	53.41	23.71

represented in terms of cost. Finally, the minimum value of objective function is to reduce the construction cost as much as possible. To tackle the limitations of conventional optimization solvers, PSO, which is available without the derivatives of equations, is employed. This means that the optimal solution can be investigated even if the mathematical model is very complex and highly nonlinear. To verify the efficacy of the proposed algorithm, two cases of EO plant are tested and the results show that the proposed algorithm provide a reasonable benefits. It is expected that the proposed algorithm would contribute to reduce the costs and find the optimal layout of various compact multi-floor processes such as Floating Production and Storage Offloading.

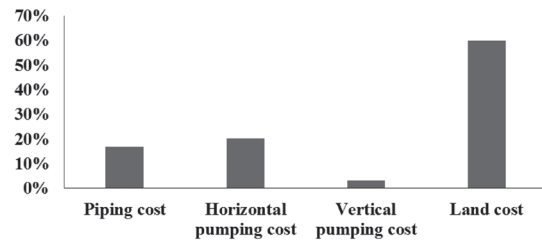


Fig. 7. The ratios according to piping cost, horizontal pumping cost, vertical pumping cost and land cost.

## Nomenclatures

$i, j$ : Process equipments,  $i$  and  $j$

$O_i$ : Binary parameter for orientation of  $i$

$a_i, b_i$ : Length of each side of  $i$

$l_i$ : Horizontal length of  $i$

$d_i$ : Vertical length of  $i$

$V_{ik}$ : Binary parameter. If  $i$  is installed on the  $k$ -th floor, 1. Otherwise, 0.

$F_{ij}$ : Binary parameter for comparing the floors of equipments  $i$  and  $j$

$C_{ij}$ : A parameter for comparing for safety and maintenance distances

$S_{ij}$ : Binary parameter. 1 if  $C_{ij}$  is positive.

$ES_i$ : Maintenance space for equipment  $i$

$x_i$ :  $x$  coordinate of center of  $i$

$y_i$ :  $y$  coordinate of center of  $i$

$z_i$ :  $z$  coordinate of center of  $i$

$m$ : Minimum safety distance

$N$ : Total number of equipments

$NF$ : Total number of floors

$X_{ij}$ : Distance in  $x$ -axis between the center of  $i$  and  $j$

$Y_{ij}$ : Distance in  $y$ -axis between the center of  $i$  and  $j$

$H$ : Floor height

$H_i$ : The height of a floor where  $i$  is installed

$U_i$ : The total height of  $i$  including  $H_i$  and  $z_i$

$U_{ij}$ : Distance in  $z$ -axis between the center of  $i$  and  $j$

$TD_{ij}$ : Total rectilinear distance between  $i$  and  $j$

$CC_{ij}$ : Piping cost per unit distance

$CV_{ij}$ : Vertical pumping cost per unit distance

$CH_{ij}$ : Horizontal pumping cost per unit distance

$X_{max}$ : Length in  $x$  direction of process site

$Y_{max}$ : Length in  $y$  direction of process site

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